Examining the Influences of the Mathematics Teachers' Pedagogical Content Knowledge on Learning Algebra in Grade Six

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Abstract – This mixed method study findings indicate that the pedagogical content knowledge (PCK) of the teachers is an influencing factor of learning algebra in grade six in Sri Lankan schools. Reliable and validated two questionnaires followed by 24 of eight mathematics teachers' classroom observations and post- lesson interviews were administered for collecting data in the explanatory sequential mixed methods research design. Field notes, lesson video recordings and the interview data were triangulated in terms of the reliability of the qualitative data while the two questionnaires were piloted and designed by Delphi method before collecting quantitative data. The Cronbach's alpha of the questionnaire-2 was 7.59. SPSS statistical software was used in the first phase while the reflexive thematic analysis method was used in the second phase of the data analysis process. The study results of the first phase indicate that the mathematics teachers' PCK in teaching algebra is at a medium level and not satisfactory. Teachers' PCK in algebra was found to be multi-functional, indicating no positive relationship with academic qualification, professional qualification and professional experiences. Further analysis showed that there was no correlation between the PCK and the PCK knowledge dimensions, declarative, procedural and conditional knowledge. Qualitative study results revealed that the mathematics teachers' PCK is low, in building algebraic concepts in the students' mind, addressing the students' algebraic misconceptions and understanding of students' algebraic thinking. Finally, learning algebra in algebraic symbols, variables and algebraic expressions were observed and found to be less effective. Therefore, content knowledge and pedagogical knowledge based professional teacher development programs are recommended for enhancing the students' learning algebra in the classrooms.

Keywords: Algebra, Algebraic Misconceptions, Algebraic Thinking, Concept Building, Pedagogical Content Knowledge

1. INTRODUCTION

Mathematics has been regarded as a major subject in the school curriculum of Sri Lanka. In the global view, there is no modern technology or modern society without mathematics (*Yemi et al.* 2013). The applications of traditional views focused mathematics knowledge was challenged by the present society since, the technical knowledge and skills based on mathematics were needed for job opportunities elsewhere. The school mathematics curriculum was needed to be reformed with new standards, converting the aspect, "knowing mathematics" into "doing mathematics" (*NCTM*, 1989). But, its practical orientations and their outcomes have still not met the targets and most of the school leavers have not been qualified for jobs that they are interested in. It means, though the mathematics competence was targeted to be developed from the past, it has still not reached the target. Algebra is a subject component in mathematics accepted as fully abstract in nature. *Kilpatrick* (2009) emphasizes the importance of algebra in the mathematics curriculum by highlighting National Mathematics Advisory panel report (2008). To improve the mathematics competence, algebra is identified as a major

subject component in the mathematics curriculum. "The conception of mathematics held by the teacher may have a great deal to do with the way in which mathematics is characterized in classroom teaching" (*Dossy*, 1992) and the teachers Pedagogical Content Knowledge (PCK) directly influences on the students' understanding (*Hill et al*. 2005). In the Shulman's classification of teachers' knowledge, PCK was described as "special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding" (*Shulman*, 1987, P.8). *An, Kulm* and *Wu*, 2004 explains PCK deeply in terms of students' learning that the different sub components influence on the contextual mathematics learning. "Teaching can be seen as either a divergent or a convergent process. A convergent process of teaching focuses on knowing students' thinking, which consists of four aspects: building on students' mathematical ideas, addressing students' misconceptions, engaging students' in mathematics learning, and promoting students' thinking mathematically" (*An et al*. 2004). The shallow knowledge of mathematics teachers especially in algebra affects the students' achievement and their mathematics performances negatively (*Odumosu et al*. 2018). Algebra teachers' performance in the learning teaching process was identified as dualistic or multiplicity in teaching behavior and in the utilizing of teaching strategies. They have not equipped with exact algebraic conceptions. It would badly influence on students' learning algebra (*Dossy*, 1992 cited in *Kesler* 1985). Knowledge of the history of mathematics and its development provide the teachers with an awareness of understanding the learner needs, learning strategies and a field of investigation from which they can have a better understanding, the learner difficulties and misconceptions (*Osei*, 2000).

Kilpatrick (2009) pointed out the National Advisory panel report (2008) and stated that the board was asked to make recommendations on the critical skills and the skill progressions for students to acquire competence in algebra and readiness for higher levels of mathematics based on the scientific evidences. Those views recommend the importance of algebra as a basic component in the mathematics curriculum. Moreover, *Simsek* and *Boz* (2016) scientifically found that the most of the researches on mathematics PCK have been carried out researches in the context of algebra. When the mathematics teachers introduce mathematical concepts in the classrooms, they should understand the definite procedures particularly interact with them and should plan relevant activities. In this process of mathematics teaching, teachers' role is very specific and it directly impacts on students' conceptions (*Greeno* and *Sande*, 2007). Often times, understanding of mathematical concepts has been focused in scientific studies, since it is not yet precisely identified by the scholars. Psychologists in the 20th century focused their attention to discover the epistemological constraints specific to mathematics learning and mathematics understanding. *Duval* (1999) analyzes and presents that *Piaget* (1961) accepted the difficulty of understanding what "intuition", a way of understanding which has close links with representation and visualization (P.3). In his views, there are many distinguished forms of mathematical intuition. Symbolization the empirical activities and manipulating them for problem solving are barely easy for the mathematics learners (*Duval*, 1999). Especially in abstract algebra, students' conception is idiosyncratic and students frequently struggle with the conception and understanding the basic concepts (*Osei*, 2000). In the history of mathematics, it is identified as a productive approach to develop conceptual understanding of algebra by understanding of both numeric and algebraic structures (*Liebenberg et al*, 1978, *Manandhar* and *Sharma*, 2021). In algebra, the letters are used in variety of aspects and those applications are distinct with each other and it makes difficult to understand "unknown" and "variables" properly (*Kuchemann*, 1978). According to *Shulman* (1986), "teachers' PCK goes beyond the knowledge of the subject matter

per se to the dimension of subject matter for teaching" (P.9) and it includes powerful components of developing the students' creativity and the problem-solving abilities. Majority of the mathematics teachers' competence in explaining algebraic concepts and developing the students' algebraic thinking is narrow (*Guler* and *Celik*, 2021). The researchers suggest that the mathematics teachers' PCK should be developed in terms of using appropriate teaching strategies and activities based on the real life for the successful learning of algebra in the classroom (*Guler* and *Celik*, 2021). The students' learning strategies are different and the teachers should be aware of the distinct learning strategies for addressing the different students' learning difficulties (*Manandhar* and *Sharma*, 2021). Mathematics teachers' PCK in teaching basic algebraic concepts at the acquisition stage is not adequate (*Chic* and *Harris*, 2007).

Research questions:

1. What is the extent of secondary level mathematics teachers' PCK in Algebra?
2. What are the factors that influence the mathematics teachers' PCK?
3. How does the teachers' PCK effect the students' learning of algebra in grade six?

2. METHODOLOGY

The scholars argue that the constructed knowledge in the teachers' mind should be measured explicitly in the classroom interactions and the teachers' cognition with intelligent understanding is appropriately measured quantitatively (*Shulman*, 1986, 1987; *Hill et al*. 2005; *Park* and *Oliver*, 2007). The researchers recommend quantitative research methodologies to measure the teachers' PCK competence and the content related intelligence before going to the classrooms but the real PCK is enacted throughout the teaching learning process, from planning the lesson to the assessment and the reflection of the outcomes. PCK interactions differ in the variants depending on the context and it is multi-functional (*Park* and *Oliver*, 2007). Researchers found that the mathematics teachers PCK is multidimensional, and it is not reliable to measure only with the use of mono instrument (*Buschang et al*. 2012). Therefore, the research methodology of this study underpinned the theoretical perspectives of *An, Kulm* and *Wu* (2004) and the philosophical foundation of pragmatism in which the researcher could minimize the subjectivity by using multiple strategies for data collection and data analysis (*Mertens*, 2007, *Frankle, Wallen* and *Hyun*, 2012). The methodological assumptions of pragmatism lead the researcher towards the mixed methods approach, understanding with the use of multiple data collecting instruments and data analyzing strategies (*Lee*, 2000). Thus, this study was launched under the mixed methods approach intending to explicitly examine the mathematics teachers' PCK and the classroom enactment in relation to algebra. According to the Sequential Explanatory research design, first phase was a quantitative study and the second was qualitative (*Creswell*, 2007).

2.1 Participants

Galle education zone was chosen as the research field considering the easy accessing facilities. Since the mathematics teachers PCK is a diverse phenomenon and depends on the teachers' intelligence and understanding, it is identical with the teachers' skills and perspectives as well (*Shulman*, 1987). Therefore, for getting an overview idea of mathematics teachers' PCK in algebra, researcher decided to provide the questionnaire-1 to all the teachers listed in Table 1. Secondly, chose a representative sample for the questionnaire-2, based on the first step result. Both questionnaires were prepared by Delphi method. According to the designing aspects of the instruments, questionnaire- 1 was prepared, aiming to get the overall teachers' PCK. Therefore, it was given to all the teachers listed in the Table 3: Participants in the second step Galle education zone. According to the results of questionnaire-1, three levels were

identified as high, medium and low, on the basis of their knowledge of PCK (Table 2). The participants of the questionnaire-2 were selected by using the stratified sampling methods from low, medium and high levels of PCK strata. The sample for the first step of the quantitative study (for questionnaire-1) was two hundred ninety-three (N=293) (Table 1) while 71 teachers were chosen (for questionnaire- 2) for the second step of the quantitative study (Table 3) representing 25% from each stratum.

Table 1: Secondary level mathematics teacher information (source: Mathematics teacher information database- 2018).

Education division	No. of Mathematics teachers
Galle	129
Akmeemana	64
Baddegama	41
Habaraduwa	59
Total	293

Table 2: The levels of mathematics teachers' PCK

PCK Level	Range of PCK marks, entitled
High	Over 50
Medium	Between 30 and 50
Low	Less than 30

Table 3: Participants of the second step

Level of the general PCK knowledge	Number of participants
Low	2
Medium	58
High	11
Total	71

The participants of the qualitative study (second phase of the study) were also chosen by stratified sampling method. Since the number of teachers in each stratum was imbalanced, the sample was selected parallel. The representative sample of the qualitative study (N=8) is showed in Table 4.

Table 4: Sample of the qualitative phase

PCK level	No. of Participants
Low	2
Medium	4
High	2
Total	8

Moreover, the research participants (secondary level mathematics teachers) represents all types of schools, 1AB, 1C and Type 2 while City, suburb and rural schools are entitled with Girls, Boys and Mixed schools.

2.2 Data collection Instruments

The questionnaire-1 included the general knowledge of PCK based on the grade 6 to 11 curriculum while the questionnaire-2 included the PCK knowledge dimensions (*Juttner et al*. 2013) and it shows in the Table 5.

In the qualitative study, classroom observations and post-lesson interviews were used to collect data from 24 grade six algebra lesson observations, three from each participant covering the topics, algebraic symbols, variables and algebraic expressions.

Table 5: Knowledge dimensions of the theoretical frame work for measuring the algebraic PCK (modified from *Juttner* et al. 2013).

PCK knowledge dimensions	PCK components
Declarative Knowledge	Knowledge about the students' concept building and understanding
Procedural Knowledge	Knowledge about algebraic teaching strategies
Conditional Knowledge	Knowledge about the students' misconceptions

2.3 Validity and reliability of the instruments

Questionnaire-1 was prepared with the help of five subject specialists and it followed the format of *Juttner et al*. (2013). Two questionnaires were prepared with the help of a

blue print. Marks for the open-ended questions in the questionnaire-1 were awarded by using a marking scheme which was prepared by the subject specialists to maintain inter-rater reliability (*Muijs*, 2004). Both questionnaires were piloted and corrected. The Cronbach's alpha of the questionnaire-2 was 7.57 by excluding one item, resulted a good internal consistency of the tool for credible results in terms of the relationships among the PCK knowledge dimensions. Different data collection instruments, observation notes, video recordings and interviews were used, and the data triangulation caused to enhance the reliability and the validity of the research (*GesNewsome et al.* 2017, *Fraenkel et al*. 2012 *Zohrabi*, 2013).

2.4 Data collection procedure

In the first phase, the questionnaire-1 was given to the mathematics teachers (N=293) for collecting data with the permission of the Galle zonal director and the zonal mathematics director. Thereafter, questionnaire-2 was shared among the selected sample (N=71). Data collection of the qualitative study was carried out in the grade six classrooms. The selected sample (N=8) represented Girls schools, Boys schools and mixed schools while they were enlisted to the rural, suburb and city schools. Three grade six algebraic lessons of each teacher in the topics of algebraic symbols, variables and algebraic expressions were observed. The observation notes, video recordings have been maintained. After each lesson observation, a semi-structured interview was conducted to clarify the observations precisely.

2.5 Data analysis process

The data collected from the questionnaire-1 and questionnaire-2 was analyzed by using the SPSS statistical software (version 25), in order to find the mathematics teachers' general PCK in algebra and the relationships between the academic qualification, professional qualification and the teaching experiences. The qualitative data were analyzed under the thematic analysis method, by using Reflexive Thematic Analysis (Reflexive TA) approach of *Braun* and *Clarke* (2019) Reflexive TA approach is not based on positivism (quantitative), it is based on interpretivism. In this approach the corded data were interpreted into their meaning and identified the themes in broader patterns of their meaning (*Braun* and *Clarke*, 2019; *Nowell*, 2017). Three themes were identified as building on algebraic concepts in the students' mind, addressing the students' algebraic misconceptions and the understanding of students' algebraic thinking.

3. RESULTS AND DISCUSSIONS

In response to questionnaire-1, secondary level mathematics teachers' PCK in algebra did not indicate sound results. Only 15.7% of the teachers possessed considerable knowledge in PCK and their scores ranged from 50 to 64. To assess the mathematics teachers' PCK in the abstract algebra, all together 16 items were administered in the questionnaire-1 including basic algebraic measures. Majority of the teachers represented the medium level, receiving scores between 30 and 50 (Table 6).

The mean and the standard deviation values of the teachers' PCK was 43.33 and 6.99 respectively (Table 7).

The overall result shows that the mathematics teachers' PCK in algebra is at a medium level. Since the test items were focused on the algebraic PCK knowledge dimensions, the teachers' knowledge and understanding of the declarative, procedural and conditional knowledge dimensions in algebraic abstract concepts viewed have not been satisfactory. According to the above results and the definitions of knowledge dimensions (Table 5), mathematics teachers do not possess a sound pedagogical content knowledge for introducing new algebraic concepts in the teaching learning process with regard to learning algebra. They are not aware of organizing and utilizing appropriate teaching strategies for teaching algebra in the classrooms. Moreover, the results revealed that the mathematics teachers do not

Table 6. PCK frequency statistics of the knowledge levels

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Low	7	2.4	2.5	2.5
	Medium	230	78.2	81.9	84.3
	High	44	15.0	15.7	100.0
	Total	281	95.6	100.0	
Missing	System	13	4.4		
Total		294	100.0		

Table 7: The standard deviation and the mean of PCK Statistics

Total		
N	Valid	281
	Missing	13
Mean		43.33
Std. Deviation		6.990

possess an adequate knowledge for addressing the students' misconceptions. The results of the questionnaire-2 indicate that the correlation of the two dimensions were not significant at the 0.01 level (Table 8). It shows that there is no considerable relationship between any of the two knowledge dimensions of PCK in algebra.

The results show that there is a very weak correlation coefficient factors in the knowledge dimensions determining the interactions of the mathematics teachers' PCK knowledge dimensions that are multi-functional. It revealed that introducing algebraic abstract concepts with the use of appropriate teaching strategies were very rare in the mathematics teachers' career in the classroom teaching and their understanding of pedagogical strategies in terms of abstract concepts is very poor. Conditional knowledge in the mean, shows the abilities of addressing the students' algebraic misconceptions (*Buschang et al*. 2012), and the medium level of PCK scores asserts that the mathematics teachers' abilities of addressing the students' misconceptions in algebra is not satisfactory. The data collected from the first part of questionnaire-1 were analyzed to find the relationships among the mathematics teachers' academic qualification, professional qualification and the professional experience with their PCK scores. It revealed that the mathematics teachers' PCK in algebra has not been developed by acquired experiences in classroom teaching (Table 11). Both academic and professional qualification show very low relationship with PCK (Table 9 and Table10). The results provide evidences to prove the nature of the multi-functional behavior of mathematics teachers' PCK in teaching algebra. It is not directly depending on the mathematics teachers' educational qualification or professional qualification or not on the professional experiences.

The second phase of the study was a qualitative study and the data collected from the sample (N=8) were analyzed using the reflexive thematic analysis approach. The summary of qualitative study is discussed under the main three themes with emphasized examples which reflected and interpreted the results of the whole study. The content of the subject matter entitled three algebraic topics in the grade six mathematics curriculum of Sri Lanka, Algebraic symbols with known and unknown, variables, and constructing of algebraic expressions.

3.1 Building on Algebraic concepts in the students' mind

The majority of the eight mathematics teachers observed, were found to be less knowledgeable about how to introduce new abstract concepts in algebra accurately for grade six students to understand it well. The teachers treat abstract concepts similarly to concrete concepts. Seven teachers out of 8 used the facts given in the text

Table 8: The correlation among the PCK knowledge dimensions

		PCK	Declarative knowledge	Procedural knowledge	Conditional Knowledge
PCK	Pearson Correlation	1	.659**	.711**	.541**
	Sig. (2-tailed)		.000	.000	.000
	N	71	71	71	71
Declarative knowledge	Pearson Correlation	.659**	1	.097	.166
	Sig. (2-tailed)	.000		.419	.167
	N	71	71	71	71
Procedural knowledge	Pearson Correlation	.711**	.097	1	.140
	Sig. (2-tailed)	.000	.419		.245
	N	71	71	71	71
Conditional Knowledge	Pearson Correlation	.541**	.166	.140	1
	Sig. (2-tailed)	.000	.167	.245	
	N	71	71	71	71

**. Correlation is significant at the 0.01 level (2-tailed).

Table 9 Correlation statistics- educational Qualification and PCK

		Educational qualification	Total
Educational qualification	Pearson Correlation	1	.042
	Sig. (2-tailed)		.482
	N	281	281
Total	Pearson Correlation	.042	1
	Sig. (2-tailed)	.482	
	N	281	281

Table 10 Correlation Statistics- Professional Qualification and PCK Correlations

		Professional qualification	Total
Professional qualification	Pearson Correlation	1	.045
	Sig. (2-tailed)		.449
	N	281	281
Total	Pearson Correlation	.045	1
	Sig. (2-tailed)	.449	
	N	281	281

Table 11: Correlation statistics- Professional Experience and PCK

		Service	Total
Service	Pearson Correlation	1	-.096
	Sig. (2-tailed)		.109
	N	281	281
Total	Pearson Correlation	-.096	1

book directly for introducing algebraic symbols. They used letters in the Sinhala and English alphabet, numbers in mathematics and the symbols of mathematical operations as examples which were taken from the grade six text book for introducing symbols. After that, differentiating the known and unknown, algebraic symbols were introduced to denote unknown and the symbols of unknown were introduced as lower case English letters. Though they need to connect the familiar concrete concepts and examples, bridging up to the abstract concepts, they were unable to unpack the explicit conceptualization. At the transition

stage, their way of shifting to the abstract from concrete were not well coordinated. Teacher number 6 was badly addicted to the text book culture and it is shown that having the text book in her hand in every second and, white board and the marker were the only resources used in introducing algebra. During the introduction of algebraic symbols, these mathematics teachers applied less productive strategies in transiting from concrete concepts to abstract concepts. They used concrete examples but, did not properly coordinate the examples for shifting into the abstract concept. Only two teachers (T5, T8) used road signs and other symbols from the real-life experiences as a way of familiarizing the symbols in the actual mean. The students actively participated and they seemed to be very interesting in learning algebra. When the teachers used mechanical shift in their strategies, the students seem to be not familiarized with the abstract concepts and feeling strange. At the end of most lessons, the students complained about their lack of understanding about the newly learnt concepts. But the teachers did not use any alternatives than ignoring their responses or repeating the same explanation over and over. It reveals that the teachers' limited PCK restricted the students' proper understanding and precise concept building. Six teachers except two (T 5, T 8) used activities and exercises directly taken from the text book, and, none of any interest or self- created activities used. As a result, the students were not provided with opportunities for enhancing their creativity. Simultaneously, the teachers showed lack of awareness in the content of algebra. Some of the teachers observed (T 1, T7), were teaching algebra without pre-preparation. Teacher1 omitted to introduce new term algebraic symbols in the first algebraic lesson even the topic is algebraic symbols. Teacher7 introduced variables and provided the text book activity. By doing the activity, one student asked "what a constant is?" At the moment, he explained it saying "Oh! I'll explain it now". In the post- lesson interview both teachers said that they had forgotten those terms to introduce at the correct time. It resulted in the lack of teacher pre-preparation to the algebraic lessons which caused for uncertainty of concept building in the students' mind. In the introduction of variables, most of the mathematics teachers used incomplete definitions or irrelevant words and word phrases resulting in less students' understanding on variables. Teacher2 did not much concern about the concept building process of variable. For an instance, He explained that the price of a coconut in the market take, Rs.40, Rs.42, Rs.38 and Rs.45 and it takes different values (text book example). Additionally, the time taken by a car for a certain journey in different days is different and the number of days in the month of February is 28 and 29. Through those examples, he built up the definition as, "A thing that varies time to time and changing the value is called a variable. As we don't know the value, we use a simple letter". It was an incomplete definition that the students were unable to construct a proper concept in their mind. In the same situation Teacher2 emphasized that "if something varies, it is a variable". Utilization of incomplete definitions, irrelevant words and word phases emphasized the lack of content knowledge in the algebraic basic concepts. In the introduction of algebraic symbols, teacher 2 attempted to describe "known" and "unknown" being in a rigid view of the conceptual mean. He frequently used the words "quantity", "constant", "varies", "can count" and "same message" in his explanations. Instead of building the abstract concepts little by little beginning from familiar concrete concepts, he attempted to convince the concept related facts and words to be permanently fixed in the students' mind. It was proven by giving similar type of number of exercises to be practiced. However, Teacher 2 showed lack of consideration about the students' conceptual shift from concrete to abstract by allowing the students to practice the concepts with more similar examples and reading by heart. Instead of that he attempted to familiarize the concept with the use of concept related words. It revealed that the mathematics teachers attempt to use concrete concepts at the students' transition stage but, they do not apply appropriate strategies for incorporating them

with the abstract concepts. Teacher8 always attempted to conduct algebraic lessons with her own approach without reflecting text book activities. She introduced variables discussing the following report (Table 12) on foreign currency exchange rates, prepared from the central bank statistics.

Table 12. The exchanging rates of a US Dollar

Date	Value in Rupees/ Rs
Sep 20	199
Oct 01	200
Oct 06	199
Oct 16	201
Oct 22	202
Nov 05	200

She used the above statistics to point out the variations of the Dollar rates increasing and decreasing time to time, and, said "it does not exist in a constant value during the above period of time". But, for the approach she continued with another activity taking more time without introducing the "variables" with the use of early discussion. The second activity was to fill up a work sheet by weighing 15 mangoes with different weights and spent 15 minutes for that activity. Altogether 21 minutes were spent only for the introduction of variables. As a result, she was unable to effectively implant the new concept in the students' mind. Though the experiences are identified very important at the stage of concept building, the teachers do not coordinate the activities systematically for assimilation of the abstract concept. Since, the teacher8 paid attention for experiencing the practical knowledge not the conceptual knowledge-based experiences. It reveals that the teachers are not aware of building abstract concepts properly in the students' mind with the help of concrete examples. We observed that mathematics teachers' PCK in terms of conceptual mean and pedagogical skills, are very poor and they do not incorporate their conceptual knowledge and the pedagogical skills effectively for accurate algebraic concept building. Further, it was asserted that all the eight teachers used mathematical expressions as concrete examples to introduce algebraic expressions but, the students claimed about their lack of understanding, because the concrete examples had not been properly used for bridging the gap between concrete and abstract concepts. The teachers attempt to automatically shift into the algebra form arithmetic without any coordination, resulted in the less understanding and indeterminacy of concept building on algebraic expressions in the students' mind. According to Piaget, students build up concepts in their mind with the help of schemas and schemas are developed with the use of inherent mental structures (every child is born with a certain cognitive structure). According to Piaget, new concept building is a mental process with assimilation, accommodation, equilibration and adaptation (*McLeod*, 2018). In the process of building algebraic concepts at the acquisition stage in grade six, the students must use the schema which is developed in arithmetic and should be well incorporated systematically, for adapting the abstract algebra. According to the steps *McLeod* (2018) presents by referring to Piaget's theory can be explained as,

- Perception and understanding of examples of algebra with the help of arithmetic understanding (assimilation).
- Thinking of a new idea by revising, changing or rebuilding schemas about abstract concepts and incorporating the previous and current schemas (accommodation).
- Dealing with more abstract concepts for familiarizing without any hesitation (equilibration).
- Storing permanent and accurate schemas in the mind about abstract concepts (adaptation).

According to the above psychological aspects, though the mathematics teachers use concrete examples of numeric sense they do not understand the systematic way of abstract concept building in the students' mind. As a

result, students' less understanding in algebra was very often observed from this study.

3.2Addressing the students' algebraic misconceptions

The most common error in algebra was that constructing algebraic expressions with subtraction. In the classroom observations, seven teachers, except teacher 8 explained the statements in the question over and over several times repeating the early expression as a remedy, it determines that the teachers do not have any alternative strategy for correcting the students' errors in constructing algebraic expressions with subtraction. Most of the time, the teachers do not attempt to diagnose the causes for the students' algebraic errors. It interprets that either the teachers are not concerned about the students' errors in algebra or they do not have required abilities for identifying them. Additionally, the teachers stand in a rigid state of accepting the accuracy of their own PCK and the content knowledge they possessed. After one of the lessons in algebraic symbols, Teacher1 asked the students "what is unknown?" One student answered, "An English letter teacher" Then the teacher responded "remind, remind, remind, it is a constant value that we don't know the exact value." She repeated her early definition for the student's error as a remedy. Her response in that situation highlighted the ignorance of the students' misconception and the inappropriate remedies applied by the teachers, to push the students towards low achievement or less interest in mathematics. She neither understood nor decided that expressing early definition only, was not sufficient to correct the student's misconception. The teacher was adamant that she has applied the most accurate and appropriate strategy with sufficient content. The mathematics teachers follow their own strategies for teaching abstract algebra in the grade six classrooms and these are the reasons to create different issues in students' understanding. With these classroom occurrences, whether the effective algebra learning occurs in the context of school curriculum is a dilemma. The following dialog shows how the Teacher1 reacts on students' misunderstandings and the students' errors in variables.

Teacher: What is the weight of the school bag?
Student P: A constant teacher
Teacher: How does it become a constant? It's a variable. It varies day by day. Isn't it?
Teacher: What is the price of a gold pound in Rupees?
Student Q: An unknown?
Teacher: Yes, it is also an unknown. But now I told you to decide whether it is a variable or a constant.
Teacher: what is the weight of one kilo of Potatoes?
Student Y: A variable teacher.
Teacher: Oh! You have still not understood this. Look here, in a one kilo of Potatoes, the number of potatoes can change but, the weight of one kilo is a fixed amount of weight. It doesn't change. So it is a constant.

In the above dialog, the teacher, attempts to correct the student's incorrect response by using the definition which is permanent in her mind throughout the lesson. She assumed that the fault is with the student. In this case the teacher was not in a position to identify the students' misconception. She takes decisions depending on her own understanding which is not always accurate. These occurrences are very sensitive for the students' expectations on mathematics education. If the mathematics teachers ignore the students' misconceptions, there is a possibility of creating frustrations on the students' mind about learning algebra. And the students' interest on algebra learning may lose.

3.3 Understanding of students' algebraic thinking

- In the classroom observations, Teacher3 followed a quite different approach to introduce the algebraic symbols. She got one purse with four pens and another with unknown number of pens and explained one can represent using a digit and another

cannot. Then she explained enough of inferences for "known". The students were allowed to provide answers. It was a good opportunity to the students for expressing their ideas. Students started to provide the parallel examples of the teacher. All the teachers except teacher 3 accepted the students' parallel answers and satisfied with the students' outcomes. But, Teacher3 motivated the students to provide different answers. As a result, the students provided very authentic examples as follows,

- The number of main management doors in a house.
- The number of wheels in a motor car.
- The vowels in the Sinhala alphabet.
- The number of colors in the rainbow.
- The number of provinces in Sri Lanka
- The number of moons in a particular planet

When the teacher appreciated the first response, most of other students also motivated to think different examples. It was a good orientation for the students' creative thinking. We observed that the most of the teachers were satisfied with the parallel examples provided by the students, limiting the students' divergent thinking and logical thinking abilities. Moreover, the mathematics teachers reacted to the questions which were asked by the students irresponsibly. It could be the reason for students to restrict the reasoning and logical thinking. The following example elicits the idea precisely.

> *Student D*: Teacher, the number of days in a year is a constant or a variable?
>
> *Teacher 5*: I'll refer and tell you tomorrow.

The students were unclear and showed their unpleasantness. In the post- lesson interview, researcher was made aware that the teacher 5 had sixteen years of teaching experiences and this text book has been used for eight years. It proves, as per the results of the quantitative study that the teachers' professional experiences are not correlated with PCK positively. Moreover, it reveals that the teacher's poor content knowledge and the lack of consideration of the accurate facts and concepts restricted the students' algebraic thinking. Additionally, it is obvious that the mathematics teachers' pre-preparation and the readiness for teaching algebra is very poor. When the teachers are involving in teaching algebra, they rarely asked questions from the students. The mathematics teachers can allow the students for reasoning and for logical thinking by asking creative questions. But, the observed eight teachers did not use the questioning as a technique for motivating the students for thinking algebraically. Most frequently we observed that the teachers asked the questions but, did not anticipate the answer from the students. Teachers provided both question and the answer. The bellow example gives evidences.

> Ex: *Teacher*: what is the number of fruits in a bunch of banana?

Students looked confused. They did not show any thrust for answering themselves. Then,

> *Teacher*: Oh! Remember now. The number of fruits is a constant value. But we don't know it exactly. If we count them, we can express it in a digit. Because, it has a fixed value.

Again the Students maintained silence. And,

> *Teacher*: Okay, shall we put "n" for it.

The students were passive listeners and they were not allowed to think and create their own answer. Reference McLeod (2018) points out the Piaget's assumptions of children's intelligence and suggests that children's intelligence, thinking and the way of reasoning are quite different to the adults. The teachers should understand the way of students' thinking and should provide opportunities for engaging in activities to develop their thinking patterns because the students possess different algebraic learning strategies (*Manandhar* and *Sharma*, 2021). If the children's reasoning and the way of thinking are needed to be understood, they must be observed by their way of thinking and from their point of view (*Mcleod*, 2018). Therefore, the study results assert that the mathematics teachers do not allow the students for thinking in

their own way thus making them less creative and not the way the algebraic thinking developed in teaching algebra in the context of school curriculum, Sri Lanka.

4. CONCLUSION

Mathematics teachers' PCK to develop meaningful transition from concrete to abstract, for learning algebra at the acquisition stage, is a dominant factor in the mathematics education (*Yildiz* and *Osdemir*, 2021). The Overall study analysis of this mixed method research points out that the mathematics teachers' pedagogical content knowledge in grade six algebra is at a medium level and it is not satisfactory in terms of students' effective understanding of algebra. Literature assert that the mathematics teachers' little awareness and narrow understanding of their PCK about teaching algebra (*Naseer*, 2018). Secondly, this research concluded that the mathematics teachers' professional experiences, academic and professional qualifications are not positively correlated with their PCK scores. The literature provides evidences for and against of the study results. The mathematics teachers' PCK is a developing knowledge category with their content knowledge and the teaching experiences (*Yusof et al*. 2021; *Lee*, 2000) and mathematics teachers' PCK represents their expertise in the process but not correlates with the number of years of experiences (*Krauss et al*. 2008). Therefore, the mathematics teachers in Galle education zone are recommended for content knowledge and PCK knowledge based professional development courses for the betterment of students' algebra learning. Align with the third research question, this study found that the students' conceptualization of basic algebra was not processed effectively in the recommended Piaget's psychological theory of conceptualization, since mathematics teachers were less knowledgeable in content knowledge and selecting less productive teaching strategies and metaphor. The teachers were unable to connect the concrete concepts and previous knowledge with the basic algebraic concepts such as symbols and variables. Therefore, the students constantly claimed about their lack of understanding of basic algebraic concepts in the classroom. Yet, the teachers were unable to respond with alternative strategies since they are not aware of remedial strategies and their lack of adequate content knowledge. *Kieran* (1992) suggests that students are struggling with the conceptualization of algebraic concepts due to the mathematics teachers' poor PCK, and it was asserted by *Yusof et al*. (2021). The qualitative analysis of the study found that the mathematics teachers were unable to introduce basic algebraic concepts, algebraic symbols, variables and algebraic expressions which are in the grade six mathematics curriculum. Further, this study results indicated that using less productive teaching strategies for shifting from arithmetic to algebra, possessing less content knowledge, utilizing incomplete definitions, applying improper examples and, using irrelevant words and meaningless word phrases that were processed in algebra teaching caused the restriction of students' proper conceptualization. The students do not identify letters as algebraic symbols, and these symbols are strange to them, letters are used as unknown and variables make it ambiguous and translating verbal expressions into algebraic expressions is difficult. Those findings assert the literature (*Yildiz* and *Osdemir*, 2018; *Kuchemann*, 1978; *Mac Gregor* and *Stacey*, 1997). Moreover, the research findings of the quantitative study found that the mathematics teachers' PCK knowledge dimensions do not have positive relationships and determine that the teachers' PCK in algebra is multi-functional. Mathematics teachers' PCK varies with the context and the teachers' professional work of PCK in the whole process of teaching and learning, interacts in multi-stylish ways (*Park* and *Oliver*, 2007).

Further, the research findings conclude that the mathematics teachers' PCK in the content knowledge is not sufficient for addressing the students' misconceptions in algebra. Repeating the same explanation over and over and ignoring the students' misconceptions were commonly

observed in students' misconceptions in algebra, indicating lack of PCK for identifying and implementing appropriate and alternative strategies in addressing the students' misconceptions in algebra. Research findings asserted that mathematics teachers are not competent to find the reasons and to make use of the relevant remedies for students' misconceptions (*Aksu* and *Kul*, 2016; *Lo*, 2020) and it has become a common deficiency in mathematics teachers' PCK.

We found that the mathematics teachers of the selected sample have been teaching algebra by limiting themselves to the text book without asking creative questions from the grade six students. It was found, less productive for algebraic concept building in the students' mind (*Black*, 2007). In this study we observed lack of technology use for teaching Algebra. Though the traditional methods of mathematics teachers' PCK facilitate the students' mathematics learning of algebra, it requires technological PCK too (*Richardson*, 2009). The study results challenged the students' rational thinking by ignoring the students' answers which created in their own way of thinking and, providing both the question and the answer without providing the opportunities for the students' creative thinking. Finally, the study results concluded that the mathematics teachers' PCK in relation to building algebraic basic concepts in the students' mind, addressing the students' misconceptions and understanding the students' thinking in algebra is not satisfactory which agrees with *Black* (2007). Interview data concludes that majority of the mathematics teachers believe, teaching Algebra is very easy for them if the student has a sound knowledge and previous experiences, yet, they are not successful in teaching Algebra (*Aksu* and *Kul*, 2016). Considering the major conclusions of this study, we recommend mathematics teachers' professional development programs both in content knowledge and pedagogical knowledge in the interest of enhancing the students' Algebra learning.

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